Dynamic Optimal Power Flow Including Energy Storage with Adaptive Operation Costs

Ju Chengquan
Energy Research Institute @ NTU
Nanyang Technological University
Singapore, Singapore
juch0002@e.ntu.edu.sg

Wang Peng
School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore, Singapore
epwang@ntu.edu.sg

***Abstract*—In past few years, the developing technology of energy storage has greatly catalyzed the evolution of energy management in power systems. The availability of storage techniques makes it possible to integrate large-scale energy storages, mitigating supply/demand variation and fluctuations brought by renewables. This paper proposes an optimal power flow (OPF) model considering energy storage with adaptive operation costs to optimize power generation and storage scheduling in multiple time periods. The objective of the proposed model is to minimize the total generation cost of generators and the operation cost of energy storages. The operation cost coefficients of energy storage are auto-adjusted according to state of charge (SOC) and time-of-use (TOU) cost of generators, specifying the dynamic characteristics of energy storage. The effectiveness of the proposed OPF model is validated by the IEEE 14-bus system that in optimal scheduling the operation of energy storage is not only price-sensitive but also responsive to the ramping load change.**

Keywords—optimal power flow; energy storage; marginal cost; optimization.

NOMENCLATURE

N_{bus}	Index of buses
c_{Gi}^2, c_{Gi}^1, c_{Gi}^0	Cost coefficients of generator at bus i
c_{Si}^2, c_{Si}^1, c_{Si}^0	Cost coefficients of energy storage at bus i
$\underline{c_{Gi}^1}$, $\overline{c_{Gi}^1}$	Min and max limits on cost coefficients of generator at bus i
$c_{Si\_ref}^2$, $c_{Si\_ref}^1$	Cost coefficients reference of energy storage at bus i
E_{Si_ref}	Capacity reference of energy storage at bus i
K_{Si}, M_{Si}	scalar coefficients of energy storage at bus i
$P_{Gi}(t)$	Output power of generator at bus i in time t
$P_{Si}(t)$	Charging/discharging power of energy storage at bus i in time t
$P_{Di}(t)$	Load demand at bus i in time t
B_{ij}	Line susceptance between bus i and j
θ_{ij}	Difference of voltage angles between bus i and j
$P_{ij}(t)$	Active power flowing from bus i to j in time t
$\underline{P_{ij}(t)}$, $\overline{P_{ij}(t)}$	Lower and upper limits of transmission line from bus i to j in time t
$\underline{P_{Gi}(t)}$, $\overline{P_{Gi}(t)}$	Output lower and upper limits of generator at bus i in time t
$\underline{R_{Gi}(t)}$, $\overline{R_{Gi}(t)}$	Ramp rate limits of generator at bus i in time t
η_{Ci}, η_{Di}	Charging and discharging efficiency of energy storage at bus i
$E_{Si}(t)$	Available capacity of energy storage at bus i at the end of time t
$\underline{E_{Si}}$, $\overline{E_{Si}}$	Lower and upper capacity limit of energy storage at bus i
Δt	Time interval (1 hour)

I. INTRODUCTION

In recent years the power grid is taking on a drastic reform. From centralization to decentralization, the transformation of physical structure has gradually blurred the boundaries between generation, transmission and distribution network. Growing dependence on fossil fuel, liberalization of electricity market and network expansion have increased the power system complexity. Furthermore, renewable energy sources, such as solar and wind energy, provide an environmental and sustainable solution to reduce fossil fuel usage and carbon emission, however their generation variability augments the operational reliability issues. Facing these problems, utility-scale energy storages have promising potentials for multiple applications in power systems, including enhancing system stability by providing additional spinning reserve, regulating voltage, frequency and power factors, as well as offering economic benefits by participating into demand response and facilitating the integration of renewable energy sources.

In face of the challenges and opportunities, conventional power systems are developing towards more intelligent, reliable and efficient entities [1], and innovative operation strategies for utility scheduling are required. Since the first optimal power flow (OPF) study by Carpentier in 1960s [2], a great number of OPF techniques have been developed over past few decades to analyze steady-state operation in power systems [3-5]. A lot of research techniques have made use of energy storage to optimize scheduling problems from different perspectives. In [6], an hourly discretized optimization method is discussed to identify the optimal daily operation for energy storage with wind turbines and hydro generation units. In [7], an AC OPF is used to study the effects of large-scale energy storage on congestion relief, emission reduction and net cost savings. In [8], A multi-period OPF with battery energy storage is proposed to investigate the effects on generation scheduling, in which the flat and time-of-use (TOU) prices of generation units are considered. In [9], the model for OPF with energy storage is defined as a two-stage finite-time optimal control problem in order to maintain consistent power on the grid. An enhanced security-constrained OPF is proposed in [10], that fast-response distributed battery energy storages are utilized to implement post-contingency corrective control actions. Several research studies focusing on OPF algorithms with energy storage have also been proposed in [11-14]. However, insufficient consideration has concentrated on two main aspects. The first one is the majority of the above OPF models plan the optimal scheduling in one isolated time period, meaning that the models can only achieve optimal results statically, ignoring the behaviors of energy storage correlated with the time. Therefore, a dynamic formulation for these optimization problems must be required. The second one is the operation costs of energy storage are neglected in most models, leading to a weakened effect of energy storage on the global optimum. These models may fail to map an overall formulation that takes energy storage into full consideration when system economic conditions change.

The motivation of this paper is to address the above problems with a dynamic OPF model including energy storage with adaptive operation costs. The remainder of the paper is organized as follows. In Section II, starting with economic dispatch and marginal cost, a scheme for adaptive operation costs of energy storage is proposed with respect to generation costs and state of charge (SOC). In Section III, a mathematical model including energy storage with cost functions for OPF is formulated, in which the characteristics of energy storage are specified. In Section IV, case studies based on the IEEE 14-bus benchmark system are investigated, and the simulation results are discussed. Finally, the conclusions are presented in Section V.

II. Adaptive Operation Costs of Energy Storage Considering TOU And SOC

Generally, the generation costs of conventional generators are described with the quadratic function as follows:

$$F(P_{Gi}) = c_{Gi}^2 P_{Gi}^2 + c_{Gi}^1 P_{Gi} + c_{Gi}^0 \tag{1}$$

The cost coefficients of conventional generators in (1) are fixed and dependent on the manufacturing specifications, maintenance scheduling and fuel prices. Recently, TOU pricing has been greatly prompted, encouraging electricity consumers to change their usage patterns in response to demand stress in peak hours. Accordingly, the electricity prices of typical generators can be determined in advance during different time periods, and their cost coefficients become time-based parameters.

Consider a single-line system, which consists of load and two generators with quadratic cost functions. The marginal costs of generators can be derived by differentiating the cost functions, which is shown in (2):

$$f(P_{Gi}) = \frac{dF(P_{Gi})}{dP_{Gi}} = 2c_{Gi}^2 P_{Gi} + c_{Gi}^1 \tag{2}$$

Within the system constraints, in order to minimize the total generation cost in the dispatch problem, the generator with low generation costs should produce as much power as possible, while the generator with higher costs should then compensate the rest power. This means the generators endeavor to operate at the same marginal cost as shown by the red horizontal lines in Fig. 1(a). If the generation cost of one generator is flat, for example, c_{G1}^2 is zero, the output power of the other generator would be restricted at the crossing point of marginal costs, as shown in Fig. 1(b).

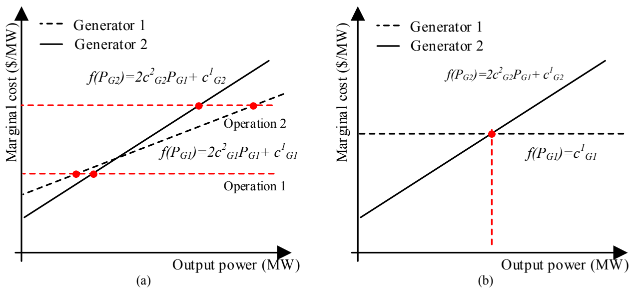

Fig. 1. Marginal costs of generators: (a) two generators with quadratic costs; and (b) one generator with the quadratic cost and the other with the fixed cost.

Let us consider another single-line system including one generator with the flat generation cost and the energy storage. In order to mimic the operational behavior of generators, the operation cost of the energy storage is set as a quadratic function similar to that of conventional generators. It should be noted that except for the loss caused by charge and discharge cycles, the energy storage should not have any practical operation costs since it does not actually produce or consume energy by themselves. Therefore, defining the virtual operation costs for energy storage is just for optimal scheduling in multiple time periods. The operation costs of energy storage can be written as follows:

$$F(P_{Si}) = c_{Si}^2 P_{Si}^2 + c_{Si}^1 P_{Gi} + c_{Si}^0 \tag{3}$$

Since the energy storage can do both charging and discharging, the operation cost of energy storage with the negative power determines the charging cost.

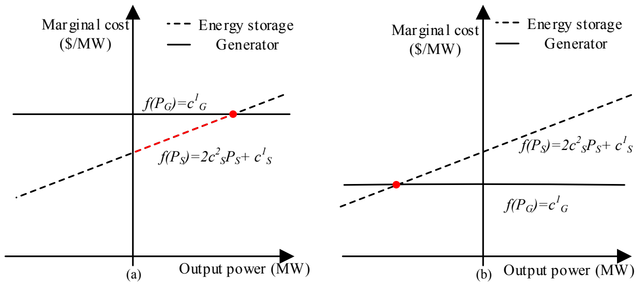

Fig. 2. Marginal costs of the generator with the flat cost and the energy storage in which the crossing point is: (a) positive; (b) negative.

Fig. 2 shows a similar principle that if the marginal cost of the energy storage is lower than the generator, the energy storage will be discharged with the power below the crossing point. When the demand is stressed, the energy storage will operate at the crossing point of marginal points. On the other hand, if the crossing point of marginal costs is on the left of y axis, then the energy storage will be charged by the generator at the crossing point in spite of the load conditions.

Expanding the above application into power system, the cost coefficients of energy storage can be adjusted based on different control variables, such as generation outputs at typical buses, load values and real-time electricity prices and so on. Therefore, setting variable marginal cost coefficients for energy storage can be used to determine the value of the exchange power with the grid. When the generation cost of the generator is time-variant, which means the marginal cost of the generator is floating up and down, the exchange power of the energy storage is then closely correlated with its cost coefficients c_{Si}^2 and c_{Si}^1.

Defining the cost coefficients of energy storage should also have to consider the SOC of energy storage in each time period as well to avoid energy storage from over-charging and over-discharging. Energy storage should be more likely to discharge than charge at high capacity, and it intends to charge when its capacity is low. Suppose the marginal cost of the generator is varying between $\underline{c_{Gi}^1}$ and $\overline{c_{Gi}^1}$, and the references of c_{Si}^2 and c_{Si}^1 are $c_{Si\_ref}^2$ and $c_{Si\_ref}^1$ at the medium SOC, correspondingly, then the cost coefficients of energy storage can be written as functions of $E_{Si}(t)$, which are shown (4) and (5):

$$c_{Si}^2 = \begin{cases} c_{Si\_ref}^2 \left[1 + \left(\dfrac{E_{Si}(t) - E_{Si\_ref}}{\overline{E_{Si}} - E_{Si\_ref}} \right)^{K_{Si}} \right], & E_{Si}(t) > E_{Si\_ref} \\ c_{Si\_ref}^2 \left[1 + \left(\dfrac{E_{Si\_ref} - E_{Si}(t)}{E_{Si\_ref} - \underline{E_{Si}}} \right)^{K_{Si}} \right], & E_{Si}(t) < E_{Si\_ref} \end{cases} \tag{4}$$

$$c_{Si}^1 = \begin{cases} c_{Si\_ref}^1 + (\overline{c_{Gi}^1} - c_{Si\_ref}^1) \left(\dfrac{E_{Si}(t) - E_{Si\_ref}}{\overline{E_{Si}} - E_{Si\_ref}} \right)^{L_{Si}}, & E_{Si}(t) > E_{Si\_ref} \\ c_{Si\_ref}^1 + (\underline{c_{Gi}^1} - c_{Si\_ref}^1) \left(\dfrac{E_{Si\_ref} - E_{Si}(t)}{E_{Si\_ref} - \underline{E_{Si}}} \right)^{L_{Si}}, & E_{Si}(t) < E_{Si\_ref} \end{cases} \tag{5}$$

(4) and (5) show that the cost coefficients of energy storage are highly correlated with the SOC and the generation cost. When the SOC of energy storage is low, c_{Si}^2 is close to $2c_{Si\_ref}^2$ and c_{Si}^1 is close to $\overline{c_{Gi}^1}$. These changing coefficients force the crossing point of marginal costs on the higher generation cost moving towards zero, limiting the maximum discharging power, while the crossing point on the lower generation cost allows the charging power to its maximum. In this way, the energy storage is able to charge with a large rate when the TOU cost of the generator is low, but constrained to discharge when the TOU cost is high. Likewise, when the energy storage is at high capacity, the adaptive coefficients will limit the charging and enable the discharging. Therefore, it can be concluded that the charging and discharging power of energy storage with adaptive operation costs can be regulated by both TOU price and SOC in the finite time horizon.

III. Mathematical Model for Optimal Power Flow

In this section, a DC OPF model based on DC power flow including energy storage with adaptive operation costs is formulated, in which the dynamic behaviors of energy storage and TOU generation costs are both considered. An adaptive cost model for energy storage is integrated to adjust and control its exchange power in multiple time periods. The model is to determine the output power of generators and energy storages, so that the total operation cost can be minimized.

A. Objective Function

With energy storage, all the OPF variables and parameters should be considered in the time domain. The objective function of the modified OPF is the minimization of the total operation cost in each time period, which can be described as:

$$f = \min \sum_{i \in N_{bus}} F(P_{Gi}) + F(P_{Si}(t)) \tag{6}$$

As mentioned in Section II, different from the generation cost of generators, the second term in (6), which represents the operation cost of energy storage, also has meaning when the output power of energy storage is negative.

B. Constraints

DC power flow extends the decoupling principle to form linear constraint sets [15]. The DC power flow equations are based on the following assumptions: the line resistance is much smaller than the line reactance; the difference of voltage angles at adjacent buses is small; and all bus voltage magnitudes are approximated as 1. Under these assumptions, the constraints for buses, generators, and transmission lines can be simplified from AC equations.

1) Bus Constraints

In DC power flow, only the active power flow equations are considered. For each bus, the power flow constraints can be written as:

$$P_{Gi}(t)+P_{Si}(t)-P_{Di}(t)=\sum_{j=1}^{N_{bus}}B_{ij}\theta_{ij},\ \ i\in N_{bus} \tag{7}$$

2) Transmission Line Constraints

Similarly, the transmission line power constraints can be written as:

$$\underline{P_{ij}(t)}<P_{ij}(t)=B_{ij}\theta_{ij}<\overline{P_{ij}(t)},\ \ i,j\in N_{bus} \tag{8}$$

3) Generator Constraints

The generator output power is restricted by its rating limits:

$$\underline{P_{Gi}(t)}\le P_{Gi}(t)\le\overline{P_{Gi}(t)},\ \ i\in N_{bus} \tag{9}$$

Since the OPF model is formulated across the time horizon, the limits on ramp rates of conventional generators should be also considered to protect generators from overload when the load change is sharp. The ramp limits can be written as:

$$\underline{R_{Gi}(t)}\le P_{Gi}(t)-P_{Gi}(t-1)\le\overline{R_{Gi}(t)},\ \ i\in N_{bus} \tag{10}$$

4) Energy Storage Constraints

The performance of energy storage at each time period can be described by two parameters: SOC, i.e., the available capacity by each end of the time period $E_{si}(t)$, and the exchange power with the system in each time period $P_{si}(t)$. Since energy storage is considered as load when it is charging, and as generator when discharging, the value of $P_{si}(t)$ can be both positive and negative. If the efficiency of energy storage is 1, the limits of $P_{si}(t)$ should be equal to the charging and discharging power of energy storage. However, considering the charging and discharging efficiency of energy storage, $P_{si}(t)$ should be limited as

$$\frac{P_{SCi}(t)}{\eta_{Ci}}\le P_{Si}(t)\le P_{SDi}(t)*\eta_{Di} \tag{11}$$

At the end of period t, the capacity of energy storage is updated based on the exchange power:

$$\underline{E_{Si}}<E_{Si}(t)<\overline{E_{Si}} \tag{12}$$

$$E_{Si}(t)=\begin{cases}E_{Si}(t-1)+P_{Si}(t)\Delta t*\eta_{Ci}, & P_{Si}(t)<0\\ E_{Si}(t-1)+P_{Si}(t)\Delta t/\eta_{Di}, & P_{Si}(t)>0\end{cases} \tag{13}$$

IV. Case Study and Results

In this section, an IEEE 14-bus system is tested to validate the effectiveness of the proposed OPF model, that in the optimized scheduling the energy storage can be both sensitive to the TOU price and responsive to the ramping load change. The parameters of this benchmark can be found in [16]. Additionally, a 100-MWh energy storage unit with the initial 30% SOC is connected to bus 1. The charging and discharging efficiency, η_{Ci} and η_{Di} , are both set to 95%. The maximum charging and discharging power are 15MW. The hourly load profile is generated by utilizing the load data in March 2014 retrieved from Energy Market Company, Singapore [17]. The optimization problem is solved using the interior point solver with MATPOWER [18] in the MATLAB environment.

A. Case I: TOU Generation Cost without Ramp Rate Limits

In Case 1, according to the load profile, the generation cost of the generator at bus 1 is considered to be the TOU price with peak, valley and flat values. Its coefficients c_{G1}^2 and c_{G1}^0 are set to zero, and c_{G1}^1 variant with time as follows:

$$c_{G1}^1=\begin{cases}2.0\$/MWh, & t=1,8\sim15,23\sim24\,h\\ 2.5\$/MWh, & t=2\sim7\,h\\ 3.0\$/MWh, & t=16\sim22\,h\end{cases} \tag{14}$$

The generation costs of other generators are presented in Table I. For the energy storage at bus 1, the coefficients of its operation cost are considered according to (4) and (5). E_{Si_ref} is given as 60% of SOC, and $c_{Si\_ref}^1$ is equal to the flat price of the generation cost as 2.5$/MWh. $c_{Si\_ref}^2$ is calculated in such a way that the charging and discharging power of the energy storage can reach their maximum at medium SOC. In this case, the value of $c_{Si\_ref}^2$ is 0.015$/MWh^2 when the SOC is 60%.

TABLE I. COST COEFFICIENTS OF GENERATORS

Bus number	Cost Coefficients		
	c_{Gi}^2 ($\$/MWh^2$)	c_{Gi}^1 ($\$/MWh$)	c_{Gi}^0 ($\$$)
2	0.025	3.0	0
3	0.001	4.0	0
6	0.001	4.0	0
8	0.001	4.0	0

The ramp rates of all generators are neglected in case I, thus the effects of ramping limits can be removed. The simulation results of the load profile, the aggregated generator output and the exchange power of the energy storage in 24 hours are shown in Fig. 3. The SOC of the energy storage in 24 hours is shown in Fig. 4.

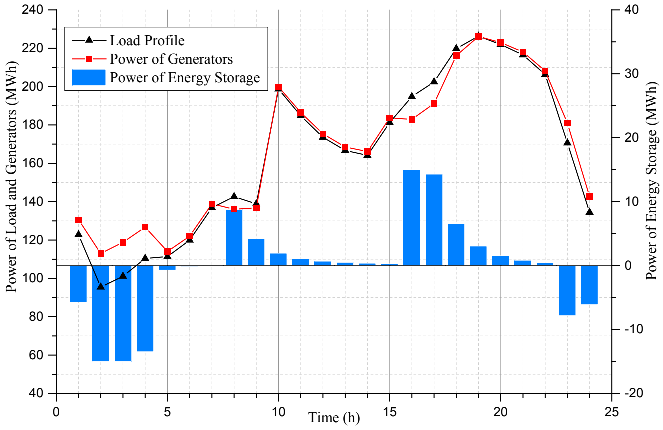

Fig. 3. Load profile, generators output and energy storage output in Case I

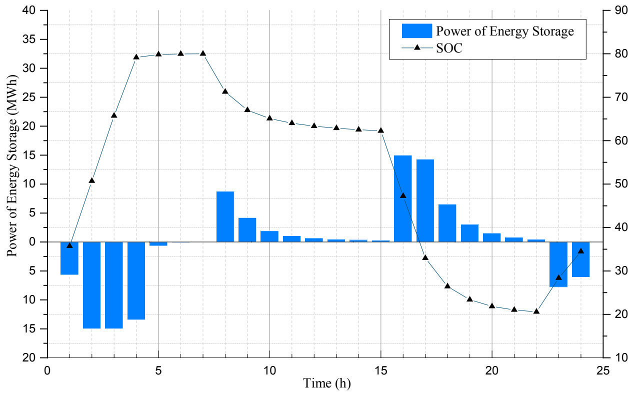

Fig. 4. Energy storage output and SOC in Case I

It is observed that the energy storage is charging when the load demand is low at hour 1 to 4. When the demand is high at hour 16 to 21, the energy storage is discharging as the generator. The reason that the energy storage is able to realize the peak shaving by following the TOU generation cost can be explained by the difference on marginal costs of the energy storage and the generator at bus 1. When the marginal cost of the generator is higher than the discharging cost of the energy storage, the energy storage will be scheduled to supply load with a cheaper price. On the contrary, when the marginal cost of the generator is lower, the energy storage will be charged with extra energy from the system, profiting from cost disparities.

It can be also seen in Fig. 4 that the maximum charging and discharging power can be regulated by both the TOU generation cost and the SOC. For example, at hour 20 and 21, although the TOU generation cost is still high, the exchange output of the energy storage has been significantly decreased, because its SOC is near the lower boundary. In this situation, the marginal costs of the energy storage and the generator are so close that the maximum output power of the energy storage is restricted.

B. *Case II: TOU Generation Cost with Ramp Rate Limits*

TABLE II. RAMP RATE LIMITS OF GENERATORS

Bus Number	Ramp Rate Limits	
	Ramp-up (MW/hour)	***Ramp-down (MW/hour)***
1	15	15
2	15	15
3	10	10
6	10	10
8	10	10

In Case II, The costs of the energy storage and generators remain the same as in Case I. However, the ramp rate limits of generators are considered, which are presented in Table II. The simulation results are shown in Fig. 5, and the comparison of the energy storage output and SOC between Case I and Case II is given in Fig. 6.

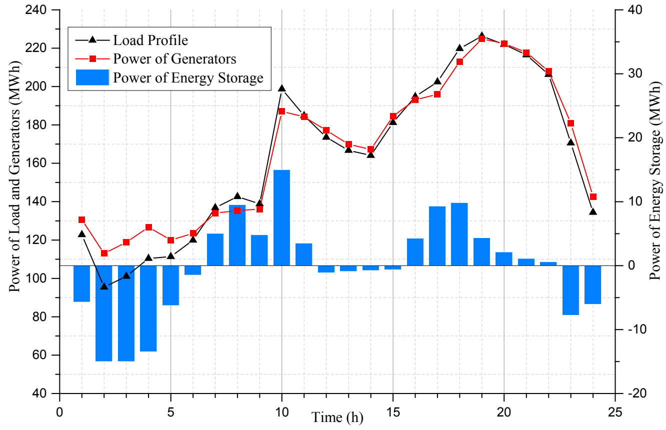

Fig. 5. Load profile, generators output and energy storage output in Case II

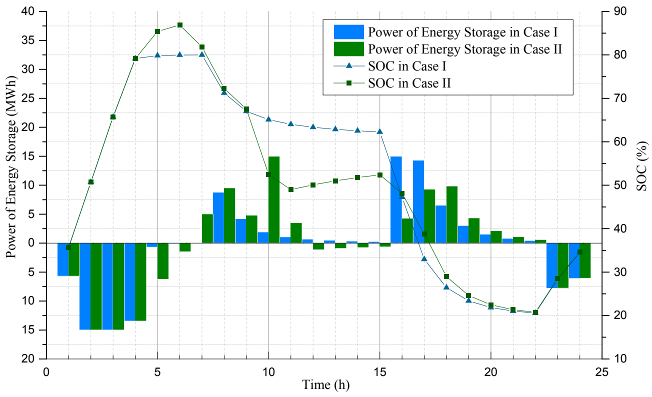

Fig. 6. Comparison of energy storage output and SOC between Case I and Case II

Fig. 5 shows the similar outcome as in Case I, that when the generators have no ramp rate limits, the energy storage can still provide peak shaving along with cost variations. Nevertheless, the differences between two cases are mainly at hour 5, hour 10, hour 16 and hour 17, where the ramp rate limits have considerable influences on the system operation. For example, in Case II, the load has suddenly increased at hour 10, however the output power of the base generator at bus 1 is restricted by its ramp-up limit. As a consequence, a part of demand surplus will be filled up by the energy storage as well as other generators with higher generation costs.

C. Case III: Fixed Generation Cost with Ramp Rate Limits

In Case III, the generation cost of the generator at bus 1 is assumed to be constant at all times, i.e., c_{G1}^2 and c_{G1}^0 are set to zero and c_{G1}^1 is 2.5$/MWh. In this way the effects of TOU generation cost are ruled out, so that the direct effect of ramp rate limits on the energy storage can be investigated. The optimization results are shown in Fig. 7.

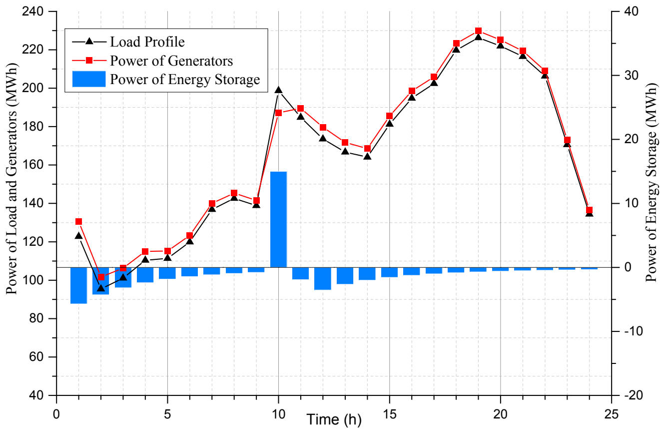

Fig. 7. Load profile, generators output and energy storage output in Case III

It can be observed that, the energy storage with the proposed operation cost can still respond to the sudden load increase in hour 10 due to the ramp-up limit of the generator at bus 1. In other time periods when the output power of generators is sufficient within network constraints, however, the energy storage does not respond to the load change since the generation cost is constant.

It is worth noting that with the fixed generation cost in this case, the generators do not reach their maximum outputs in most of the time periods. This leads to the fact that at some hours such as hour 18 and hour 23, even though the load is changing significantly, due to the transmission line constraints, however, more expensive generators rather than the base generator or the energy storage would commit to meet the demand, at the expense of higher generation costs nevertheless.

V. Conclusion

A mathematical model for DC OPF considering energy storage with adaptive operation costs is proposed in this paper to optimize the generation and energy storage scheduling in multiple time periods. The operation cost coefficients of energy storage are adjusted automatically according to SOC and TOU generation costs, specifying the dynamic characteristics of energy storage. An IEEE 14-bus benchmark system is simulated to show the effectiveness of the proposed OPF model, whose objective is to minimize the system operation cost of generators and energy storages. The simulation results validate that with the proposed method, the operation of the energy storage is not only price-sensitive but responsive to the ramping load change. It is concluded that the energy storage with adaptive operation costs can be utilized not only to meet system ramping requirements, but also to help flatten the load profile in peak hours.

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