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Optimal Power Flow with Worst-case Scenarios considering Uncertainties of Loads and Renewables

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Abstract—The growing interest on RES gives traditional power systems an opportunity to evolve towards more sustainable and environmental entities, however the viability of RES would induce stability and reliability issues in power systems. In this paper, a DC optimal power flow (OPF) algorithm considering worst-case scenarios is proposed. It accounts for uncertainties brought by loads and renewable energy sources (RES), while in the meantime the highest system reliability level can be achieved. By assigning selected values with largest probabilities to random variables, the probabilistic OPF formulation is converted into a set of deterministic OPF problems in which the additional auxiliary constraints are implemented to represent the uncertain influences. The proposed OPF with worst-case scenarios is applied into an IEEE 14-bus and 57-bus benchmark power system. The results in the simulation along with other OPF techniques shows the validity and robustness of the algorithm.

Index Terms—Optimal power flow (OPF); renewable energy sources (RES); reliability; optimization.

I. Introduction

In recent years, the integration of renewable energy sources (RES) provides a sustainable and environmental solution to reduce fossil fuel dependency and carbon emission. However, the disadvantages of large-scale power systems have been progressively revealed, that stability and reliability issues have gradually become public concerns due to long-distance electricity transmission and intermittent nature of RES. Optimal Power Flow (OPF) is considered to be a useful tool to determine the most cost-efficient operation strategy for power systems. Several OPF studies have been conducted with respect to system design and planning [1], operation [2], management [3], [4] and power flow algorithms [5]–[8]. In the early years, due to computational limits and lack of techniques, most OPF studies were based on deterministic models which may be unable to accurately present the stochastic nature of generation outputs and random loads. In addition, the implementation of distributed generation brings more uncertainties into power systems. Therefore, deterministic OPF analysis can no longer present the variability of the system accurately, and the probabilistic OPF has been developed to deal with such stochastic issues.

A number of probabilistic OPF algorithms have been proposed during past few decades. Monte Carlo simulation has been intensively used to present the intrinsic characteristics of power systems [9]. Existing research has focused on the improvement of probabilistic OPF methodologies. In [10], a two-point-estimate-method (2PEM) algorithm is applied in probabilistic OPF to address uncertainties against the competitive electricity market. A hybrid solar-wind power system is discussed in [11], in which the correlation of uncertain variables are considered by using the modified 2PEM algorithm. In [12], forecasting errors of RES is considered by using a chance-constrained multi-objective probabilistic OPF. In [13], the probabilistic OPF incorporated with Taguchi orthogonal array tables (TOAT) is proposed to account for uncertainties by RES. However, most of the existing probabilistic techniques cannot provide the full set of state spaces, which may end up with local optimal solutions. For large systems with multiple constraints at buses and trans-mission lines, these probabilistic algorithms may lead to large errors in the solution or even fail to map a global optimum. In addition, reliability issues are barely concerned in the literature in which most of the existing algorithms do not utilize the effects of uncertainty into robust operation.

To this end, a DC OPF algorithm considering worst-case scenarios is proposed in this paper that provides the minimum operating cost and maintains the highest possible reliability levels. Inspired by [13] and [14], the probabilistic OPF problem is transformed into a set of deterministic OPF formulations based on selected scenarios representing RES and load uncertainties, and additional auxiliary constraints are imposed into the scenarios to narrow down the optimal solution space. The remainder of the paper is organized as follows. In Section II, the general probabilistic OPF formulation with RES is modeled. Section III focuses on the modeling of uncertainties of RES and load, and a modified OPF considering worst-case scenarios is proposed. Case studies based on the IEEE 14-bus and 57-bus benchmark system are investigated in Section IV. in which simulation results validate the effectiveness of the proposed method by using two quantification indices.

II. Probabilistic OPF Formulation

The OPF formulation incorporating AC power flow equations is sometimes infeasible to analyze large power systems due to nonlinearity and non-convexity [15]. DC OPF extends the decoupling principle to form linear constraint sets and

requires less computation time since only active power flow equations are considered [16], [17]. DC OPF is simplified from AC OPF based on three assumptions: a) the line resistance G_{ij} is much smaller than the line reactance; b) the difference of voltage angles at adjacent buses is small; and c) all bus voltage magnitudes are approximated as nominal values. In the following discussion, we will focus on the DC OPF formulation unless otherwise specified.

A. Objectives

The objective of OPF is to minimize the total operational cost of generators, which can be formulated as follows:

$$F\left(P_{Gi}\right) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i, \ i \in N_B \tag{1}$$

where a_i, b_i and c_i are cost coefficients and N_B is the number of buses.

B. Constraints

We denote θ_i to be the power angle at bus i and B_{ij} to be the susceptance between bus i and j. The real power flow at buses and transmission lines can be derived respectively as follows:

$$P_{Bi} = \sum_{\substack{j \in N_B, \\ j \neq i}} B_{ij}(\theta_i - \theta_j), \ i, j \in N_B \tag{2}$$

$$P_{Tij} = B_{ij}(\theta_i - \theta_j), \ i, j \in N_B \tag{3}$$

The above equations can be written in the matrix form as follows:

$$\underbrace{\boldsymbol{P_B}}_{N_B \times 1} = \underbrace{\boldsymbol{B_B}}_{N_B \times N_B} \theta \tag{4}$$

$$\underbrace{\boldsymbol{P_T}}_{N_T \times 1} = \underbrace{\boldsymbol{B_T}}_{N_T \times N_B} \theta \tag{5}$$

where N_T is the number of transmission lines, $\boldsymbol{B_B}$ and $\boldsymbol{B_T}$ are corresponding supceptance matrices.

The RES penetration introduce more uncertainties into power systems that make operation conditions more unpredictable. Probabilistic OPF addresses uncertainty parameters in which traditional generators are considered as certain and controllable variables, while RES and loads are regarded as uncertain variables. Parameters of RES and loads can be modeled as variables with probabilistic distributions [18]. At each bus, the power balance can be express as follows:

$$\widetilde{P}_{Bi} = P_{Gi} + \widetilde{P}_{Ri} - \widetilde{P}_{Li}, \ i \in N_B \tag{6}$$

where the tilde superscript refers to the corresponding uncertain variables. Considering (4) and (5), we can write (6) in the matrix form as follows:

$$\widetilde{\boldsymbol{P_B}} = \boldsymbol{P_G} + \widetilde{\boldsymbol{P_R}} - \widetilde{\boldsymbol{P_L}} = \boldsymbol{B_B}\boldsymbol{B_T^{-1}}\boldsymbol{P_T} \tag{7}$$

For each transmission line, the power flow is restricted by the maximum limits. The inequality constraints for transmission lines can be expressed combining with (7) as:

$$\underline{\boldsymbol{P_T}} \leq \widetilde{\boldsymbol{P_T}} = \boldsymbol{B_T^{-1}}\boldsymbol{B_B}(\boldsymbol{P_G} + \widetilde{\boldsymbol{P_R}} - \widetilde{\boldsymbol{P_L}}) \leq \overline{\boldsymbol{P_T}} \tag{8}$$

For each generator unit, the output is limited by its power ratings, which is formulated as follows:

$$\underline{P_{Gi}} \leq P_{Gi} \leq \overline{P_{Gi}}, \ i \in N_B \tag{9}$$

C. Probabilistic DC OPF Formulation

Following the above objective function and constraints, the probabilistic OPF formulation is presented as follows:

$$f : \min \sum_{i \in N_B} F(P_{Gi})$$
$$\text{s.t. } (1), (4), (5), (7) - (9) \tag{10}$$

It can be easily observed that the probabilistic formulation in (10) is linear with random variables in (7) and (8).

III. WORST-CASE SCENARIO FORMULATION

In this section, worst-case scenarios are selected by aggregating load variables and selecting representative values of RES. The original probabilistic OPF is transformed into several deterministic OPF formulations with additional constraints manifesting uncertainties of loads and RES.

A. Uncertainties Modeling of Loads and RES

Fluctuations exist mainly in loads and RES forecasting errors [11]. The uncertainties of loads and RES are independently modeled since the correlations on their forecast techniques are usually weak. The loads are considered to follow the normal distribution [19] as follows:

$$P_{Li} \sim N\left(\mu_i, \sigma_i^2\right), \ i \in N_B \tag{11}$$

In this paper, wind turbines are assumed to be the RES in the system. Generally, the generation of wind turbines follow the Weibull distribution [20]:

$$\rho\left(v\right) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\frac{v^k}{c}} \tag{12}$$

where k and c are the shape and scale parameter, respectively. v is the wind speed. The outputs of RES can be approximated in relation with v as follows [11], [21]:

$$P_{Ri} = \begin{cases} P_{Ri\_rated}^i \dfrac{v - v_{cin}^i}{v_{rated}^i - v_{cin}^i}, & v_{cin}^i \leq v \leq v_{rated}^i \\ P_{Ri\_rated}^i, & v_{rated}^i \leq v \leq v_{cout}^i \\ 0, & otherwise \end{cases}, i \in N_B \tag{13}$$

where $P_{Ri\_rated}^i$ is the power rating of ith wind turbine, v_{cin}^i, v_{cout}^i and v_{rated}^i are the rated speed, cut-in speed and cut-out speed, respectively.

B. Aggregation of Load Variables

Let $\widetilde{P}_{L-agg}$ denote the array of the aggregation of all loads:

$$\widetilde{P}_{L-agg}\widetilde{P}_{L-agg} = B_T^{-1}B_B\widetilde{P}_L \tag{14}$$

Since the load variables at different buses are independent [22], recalling (8), the term including loads forms the following normal distribution:

$$\widetilde{P}_{L-agg} \sim N(B_T^{-1}B_B\mu, \sigma^T(B_T^{-1}B_B)^T B_T^{-1}B_B\sigma) \tag{15}$$

where μ and σ refers to the mean value and stand deviation array of the load variables, respectively.

For each transmission line, the term including $\widetilde{P}_L$ with N_B random variables in (8) is transformed into one aggregated variable representing effects of all loads. In total, the number of random variables is reduced from $N_L + N_B$ to $1 + N_B$. As a consequence, the computational burden has been alleviated.

C. Representative Values for Random Variables

The number of representative values for random variables in loads and RES also has significant influences on the computational complexity. Apparently, a smaller number of representative values lead to the faster processing time. In this paper, two representative values are selected for the aggregated load and RES variables [10], [11]. For the aggregated load $\widetilde{P}_{L-agg}$, two representative vectors are selected as follows:

$$P_{L-agg} \in \{B_T^{-1}B_B\mu + \sigma^T(B_T^{-1}B_B)^T B_T^{-1}B_B\sigma$$
$$B_T^{-1}B_B\mu + \sigma^T(B_T^{-1}B_B)^T B_T^{-1}B_B\sigma\} \tag{16}$$

For RES variables $\widetilde{P}_R$, zero and the rated capacity are adopted respectively since the outputs of RES are always in this zone. The selected values are presented as follows:

$$P_R \in \{0,\ P_{R\_rated}\} \tag{17}$$

D. Selection of Worst-case Scenarios

After selection of representative values for each random variable, a total number of 2^{1+N_B} possible scenarios are formed. Among all the scenarios, the constraints on transmission lines must be determined for each scenario. (8) can be presented as a set of the following formulations:

$$\begin{cases} \underline{P_T} - B_T^{-1}B_B P_G \leq B_T^{-1}B_B\widetilde{P_R} - \widetilde{P}_{L-agg} \\ \overline{P_T} - B_T^{-1}B_B P_G \geq B_T^{-1}B_B\widetilde{P_R} - \widetilde{P}_{L-agg} \end{cases} \tag{18}$$

Suppose the total number of M scenarios based on representative values of random variables have been selected. The constraint for each transmission line can be thus formulated as follows:

$$\begin{cases} \underline{P_{Ti}} - B_{Ti}^{-1}B_{Bi}P_G \leq \min_{m \in M}(B_{Ti}^{-1}B_{Bi}\widetilde{P_{R,m}} - \widetilde{P}_{L-agg,i,m}) \\ \overline{P_{Ti}} - B_{Ti}^{-1}B_{Bi}P_G \geq \max_{m \in M}(B_{Ti}^{-1}B_{Bi}\widetilde{P_{R,m}} - \widetilde{P}_{L-agg,i,m}) \end{cases}$$
$$i \in N_T \tag{19}$$

where the subscript i denotes the ith row of the corresponding matrix.
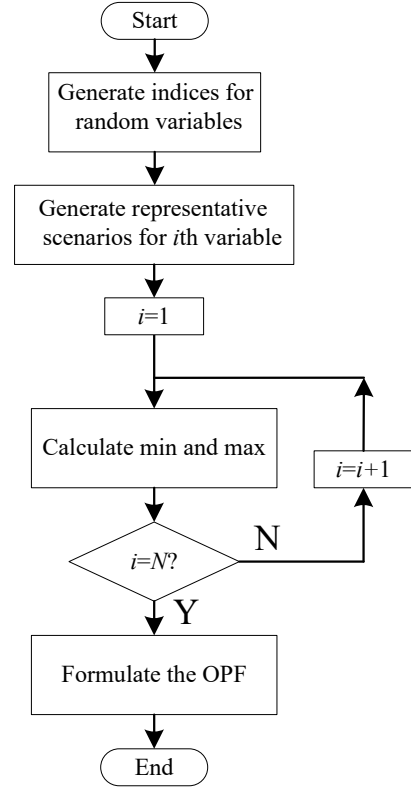

Fig. 1. Flowchart of Selecting the Worst Scenario.

The scenarios with the max/min constraints are regarded as worst-case scenarios, because they provide the strictest line constraints for the OPF that has the highest reliability levels. As all the random variables are independent, the additions of the right terms in (19) are linear and can be further written as follows:

$$\underline{P_{Ti}} - B_{Ti}^{-1}B_{Bi}P_G \leq \sum_{k \in N_B} \min_{m \in M}\left(B_{Ti,k}^{-1}B_{Bi}\widetilde{P}_{R,m}\right)$$
$$- \max_{m \in M}\left(\widetilde{P}_{L-agg,i,m}\right) \tag{20}$$

$$\overline{P_{Ti}} - B_{Ti}^{-1}B_{Bi}P_G \geq \sum_{k \in N_B} \max_{m \in M}\left(B_{Ti,k}^{-1}B_{Bi}\widetilde{P}_{R,m}\right)$$
$$- \min_{m \in M}\left(\widetilde{P}_{L-agg,i,m}\right) \tag{21}$$

The process of selecting worst-case scenarios is illustrated in Fig. 1. For random variables of RES, the corresponding terms $\max_{j \in M}\{-B_{i,k}, P_{R,k,j}\}$ are independent with other variables. Thus, the max/min worst-case scenarios of each transmission line can be obtained by selecting maximum and minimum values for each random variable and adding each corresponding terms.

E. Proposed DC OPF Formulation

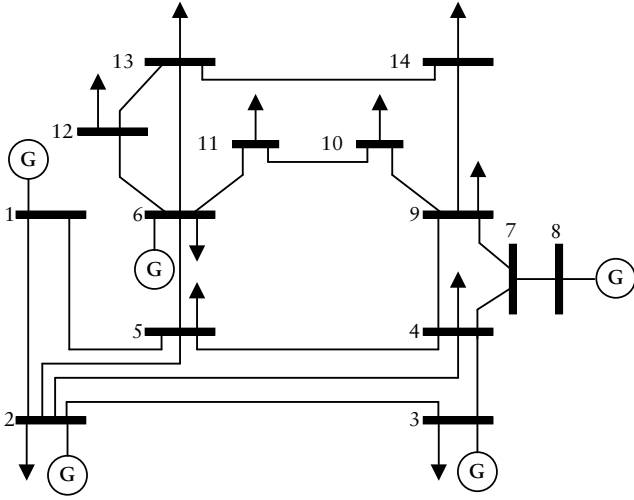The additional auxiliary constraints are added into the proposed probabilistic OPF formulation with worst-case sce-

Fig. 2. Schematic diagram of IEEE 14-bus power system.

Type	Bus number	Power limits(MW)		Cost parameter		
		P_{max}	P_{min}	a_i	b_i	c_i
generators	1	100	0	0	25	0
	2	80	0	0.25	20	0
	3	80	0	0.01	40	0
	6	60	0	0.01	40	0
	8	60	0	0.01	40	0
RES	2	30	0	0	0	0
	3	30	0	0	0	0

From	To	Resistance (p.u.)	Reactance (p.u.)	Susceptance (p.u.)	Rated Capacity (MW)
1	2	0.01938	0.05917	0.0528	80
1	5	0.05403	0.22304	0.0492	40
2	3	0.04699	0.19797	0.0438	50
2	4	0.05811	0.17632	0.034	40
2	5	0.05695	0.17388	0.0346	40
3	4	0.06701	0.1710	0.0128	50
4	5	0.01335	0.04211	0	80
4	7	0	0.20912	0	40
4	9	0	0.55618	0	40
5	6	0	0.25202	0	40
6	11	0.094989	0.1989	0	40
6	12	0.12291	0.25581	0	40
6	13	0.06615	0.13027	0	40
7	8	0	0.17615	0	40
7	9	0	0.11001	0	40
9	10	0.03181	0.0845	0	40
9	14	0.12711	0.27038	0	20
10	11	0.08205	0.19207	0	20
12	13	0.22092	0.19988	0	20
13	14	0.17093	0.34802	0	20

narios, which is obtained as follows:

$$f : \min \sum_{i \in N_B} F(P_{Gi})$$
$$\text{s.t. } (1), (4), (5), (7) - (9), (16), (17), (20), (21) \quad (22)$$

IV. CASE STUDY

In this section, an IEEE 14-bus and IEEE 57-bus benchmark system are simulated. The simulations with MATPOWER in the Matlab environment [23]. In order to show the effectiveness of the proposed method, a deterministic formulation with conventional transmission line constraints and a probabilistic OPF based on TOAT in [13] are simulated as well.

A. Validation Indices

The validation process needs to be conducted in order to appropriately compare and evaluate the performances of different methods. Monte Carlo simulation (MCS) is adopted to generate a series of random scenarios [24], since it is capable of manipulating a large number of random samples with different combinations of uncertain values [25]. Random variables of load and RES are sampled based on their distribution characteristics mentioned in Section III. Meanwhile, the generator outputs are set as the optimized results except for those at the slack bus. In each scenario generated by MCS, the generation constraints and transmission line constraints are investigated. If all the constraints are within their limits, the tested scenario will be identified as a feasible scenario; otherwise, it is identified as unfeasible [26]. After all the scenarios generated by MCS are examined, the robust degree of the optimized results can be calculated as follows:

$$d_f = \frac{n_v}{n_{MCS}} \times 100\% \quad (23)$$

where n_v and n_{MCS} are the number of all feasible and tested scenarios, respectively.

The other confirmatory parameter to evaluate the generation insufficiency is expected energy not supplied (EENS). It is defined as the average of differences between the optimized generation outputs and the variable demand values when the scenario is unfeasible. In scenarios with feasible solutions, EENS is counted as zero. In two case studies, n_{MCS} is set to be 10000. The standard deviation for the aggregated loads is set to be 5%. In addition, the RES distribution is set as the Weibull distribution with k=1.72 and c=1.91.

B. IEEE 14-bus System

The diagram of the IEEE 14-bus benchmark power system is presented in Fig. 2. The transmission network parameters are presented in [27]. In addition, two 30 MW wind turbines are added at bus 2 and 3, respectively. For all generation units, including RES and generators, their output limits and cost parameters are presented in TABLE I, in which traditional generators follow the quadratic cost formulation and RES are assumed to have no operational cost. The transmission line constraints are shown in TABLE II. There are 11 loads in

TABLE III
SIMULATION RESULTS OF IEEE 14-BUS SYSTEM

	Process time(s)	Robust degree	EENS (MW/h)	Generation cost (10^3\$/h)
Traditional	0.022249	4.29%	57.0683	**6.0715**
TOAT	0.045919	73.92%	15.2941	6.6863
Worst case	**0.028087**	**89.92%**	5.9574	6.8649

TABLE IV
OPTIMIZED GENERATION OUTPUTS OF IEEE 14-BUS SYSTEM

Type	Bus number	Generation outputs (MW)		
		Traditional	TOAT	Worst case
generators	1	100	60.4041	49.0166
	2	40.7763	40.2556	42.5230
	3	19.4078	30.6494	34.3694
	6	19.4079	35.7711	35.6472
	8	19.4078	31.9196	37.4435
RES	2	30	30	30
	3	30	30	30

TABLE V
GENERATION PARAMETERS IN IEEE 57-BUS SYSTEM

Type	Bus number	Power limits(MW)		Cost parameter		
		P_{max}	P_{min}	a_i	b_i	c_i
generators	1	200	0	0	20	0
	2	100	0	0.01	40	0
	3	100	0	0.25	20	0
	6	100	0	0.02222	40	0
	8	100	0	0.01	20	0
	9	100	0	0.01	40	0
	12	100	0	0.0323	20	0
RES	8	50	0	0	0	0
	9	50	0	0	0	0
	12	50	0	0	0	0
	43	50	0	0	0	0

TABLE VI
SIMULATION RESULTS OF IEEE 57-BUS SYSTEM

	Process time(s)	Robust degree	EENS (MW/h)	Generation cost (10^3\$/h)
Traditional	0.026893	1.20%	171.0055	**2.9095**
TOAT	0.533138	69.00%	44.8387	3.164
Worst case	**0.043113**	**93.91%**	10.9609	3.2538

the IEEE 14-bus system, which can be aggregated into one uncertain variable based on (14). Therefore, there exists three random variables including an aggregated load variable and two RES variables.

The process time, robust degree, EENS and operating cost of the traditional deterministic OPF, the probabilistic OPF with TOAT and the proposed OPF with worst-case scenarios are presented in TABLE III respectively. The optimized generation outputs of three methods are also shown in TABLE IV.

The results indicate that the traditional OPF achieves the lowest generation cost, whereas its robust degree is low compared with other OPF methods. In the traditional OPF, the generator with the least marginal cost (i.e. generator at bus 1) produces the most power in order to achieve the minimum operational cost, reaching its maximum capacity. In real situation, however, generators which have low costs do not always operate at their full capacity due to system constraints and reserve requirements. Therefore, the result in traditional OPF provides an extreme operation strategy. When the fluctuations on loads and RES occur, there is a large probability that the transmission lines are overload at the risk of failure. On the contrary, when the uncertainties of loads and RES are considered in the proposed OPF with worst-case scenarios, part of transmission capability is reserved by additional auxiliary constraints, resulting into more averaged power dispatch strategies. Therefore, the OPF with worst-case scenarios achieves higher robust degrees and lower EENS at the expense of the increased generation costs.

Compared with the proposed OPF with worse-case scenarios, the main drawbacks of the OPF with TOAT includes the decision making on scenarios selection and computational time. The TOAT method is unable to cover the strictest

conditions for three random variables include the aggregated load and two RES, since it selects three scenarios out of eight combinations. Therefore, its robust degree is less than the proposed OPF. On the other side, large computational time is consumed to produce orthogonal arrays. Hence, the computation time would be drastically reduced if the arrays are preprocessed prior to the problem formulation.

C. IEEE 57-bus System

In the IEEE 57-bus benchmark power system, four wind turbines with the capacity of 50 MW are connected to bus 8 and 9, 12 and 43, respectively. The parameters of all generation units are presented in TABLE VI. Similarly, with an aggregated load, there are five random variables including one aggregated load and four RES variables. The process time, robust degree, EENS and operating cost of three cases are presented in TABLE VI, respectively. The optimized generation outputs of three methods are also shown in TABLE VII.

TABLE VII shows a similar result that the traditional OPF achieves the lowest generation cost compared with the OPF with worst-case scenarios, whereas its robust degree and EENS is quite poor. It is also illustrated in TABLE VII that the generator with the least marginal cost (i.e. generator at bus 1) operates at its maximum capacity in the traditional OPF. In the OPF with worst-case scenarios, the generator output at bus 1 has been decreased due to the additional auxiliary constraints on the transmission lines. As a consequence, the robust degree and EENS has been increased.

In the IEEE 57-bus system, five random variables come from the aggregated load and four RES, resulting in a drastic

TABLE VII
OPTIMIZED GENERATION OUTPUTS OF IEEE 57-BUS SYSTEM

Type	Bus number	Generation outputs (MW)		
		Traditional	TOAT	Worst case
generators	1	250	150.3943	87.9669
	2	15.1642	0	0
	3	40.6065	40.5084	45.8984
	6	15.1642	80.1814	100
	8	400	400	400
	9	15.1642	99.9999	100
	12	314.7005	279.7157	316.9346
RES	2	30	30	30
	8	50	50	50
	9	50	50	50
	12	50	50	50
	43	50	50	50

decrease on the robust degree and EENS in the OPF with TOAT. In the meantime, the proposed OPF holds a high reliability level. It can be thus concluded that for the OPF with TOAT, it is very dependent on the size of random variables, whereas the proposed OPF with worst-case scenarios can always provide the strictest conditions to maintain the system reliability.

V. CONCLUTION

This paper presents a novel DC OPF algorithm that accounts for the uncertainties brought by RES and loads. With the consideration of worst-case scenarios, the highest operational reliability is provided while the traditional generators are scheduled in a cost saving fashion. In the formulation, assigning values with largest probabilities to random variables, the probabilistic OPF is transformed into a set of deterministic OPF formulations with auxiliary constraints, whose upper and lower boundaries are determined by the max/min worst-case scenarios in a finite set of selections. The proposed OPF algorithm is tested in an IEEE 14-bus and an IEEE 57-bus power system. By comparing with other methods provided in the literature, validation indices have shown the effectiveness of the proposed OPF with worst-case scenarios.

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