A Temporal Decentralized Algorithm for Optimal Stochastic Energy Scheduling in Microgrids

Chengquan Ju[1], Yi Tang[2], Yu Wang[2], and Yan Xu[2]
[1]Energy Research Institute @ NTU, [2]School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore, Singapore
Email: cqju@ntu.edu.sg

***Abstract*—This paper proposes a decentralized alternating direction method of multipliers (ADMM) algorithm for solving the optimization problem of energy scheduling in microgrids. Different from the other ADMM-based distributed approaches which decomposes the monolithic problem spatially into smaller tractable subproblems, the proposed method adopts a temporal decomposition on the scenario tree inherited from multi-stage stochastic programming. Each node in the scenario tree serves its own optimization with local variables and constraints, and iteratively updates information with adjacent nodes. By implementing the proposed ADMM algorithm in a rolling fashion, simulation results have shown the fast convergence of the temporal distribution framework, and comparisons on optimal value and computation time with other optimization approaches reveals its advantages and effectiveness.**

***Index Terms*—Microgrids, stochastic optimization, decentralized algorithm.**

I. Introduction

Intensive research on modern control technologies and increasing amount of investments on renewable energy sources (RES) have evolved conventional centralized electrical systems into smart, interactive, and distributed entities. In the context of microgrid which is comprised of local distributed generators and varying loads, the dominant adoption of wind turbines (WT) and solar photovoltaics (PV) may cause stability issues on system operation due to intermittency and non-dispatchability of RES. As bidirectional mediator, energy storages (ES) are capable of mitigating instantaneous power mismatch, conducting energy shifting and providing auxiliary services based on various control strategies.

For energy scheduling in microgrids, the system operation problem is often formulated as an optimization model to varied desired objectives using forecast information of RES, loads as well as electricity prices, such as minimization of operational costs and maximization of social benefits. Nevertheless, forecast stochasticity brings two-sided challenges on the dispatch problem. On one hand, it is impossible to make perfect predictions on future information so as to make current decisions under uncertainty, and the operational decisions are always made sequentially over time. On the other hand, it is usually intractable to solve one monolithic problem by tracking stochastic behaviors.

To this end, multi-stage stochastic programming has been intensively discussed in existing literature, in which so called *here-and-now* decisions are made in advance, then the *wait-and-see* decisions are determined upon the realization of uncertainties [1]. By using a finite number of scenarios to present uncertain randomness, an equivalent large-scale model can be formulated with a set of variables in subsequent stages.

A variety of mathematical models and approaches have been developed for solving multi-stage stochastic programs, such as progressive hedging [2], [3], model predictive control [4], [5], reduction to two stage approximation [6]–[8], and nested benders decomposition [9], [10], in which the basic idea is to transform multistage stochastic programs with a finite number of possible future scenarios into deterministic equivalents. Due to computational intractability, decomposing approaches are usually needed to tackle large-size problems. In particular, the alternating direction method of multipliers (ADMM) algorithm [11] has gained tremendous attention in recent years for solving large-size problems, and has been widely employed in statistics, machine learning and related areas in which extensive training samples are required.

A diversity of studies have applied ADMM into energy scheduling in microgrids as well as optimal power flow in distribution networks to decompose the single monolithic problem in spatial into a series of small subproblems delegated by parallel agents [12]–[14]. While ADMM is used to make *spatial* decomposition in most literature, in this paper, we propose a novel *temporal* decentralized algorithm aiming at the stochastic process in form of scenario tree. The multi-time dynamic optimization model is formulated as a multi-stage stochastic program binded with the scenario sets in a tree structure, in which each node solves the small convex optimization problem using local information and updates with adjacent nodes iteratively.

The remainder of this paper is organized as follows. In Section II, The mathematical optimization model for the microgrid is presented, and its compact form of multi-stage stochastic optimization under sampled scenarios is provided as well. The proposed distributed algorithm based on scenario tree is elaborated in Section III. Case studies and simulation results are discussed in Section IV. At last, the conclusion is summarized in Section V.

II. Mathematical Model of Microgrid

A. Formulation of Optimization Problem

We consider a general microgrid model that comprises the point of common coupling to the utility grid (UG), RES including WT and PV, microturbine (MT), fuel cell (FC), ES and

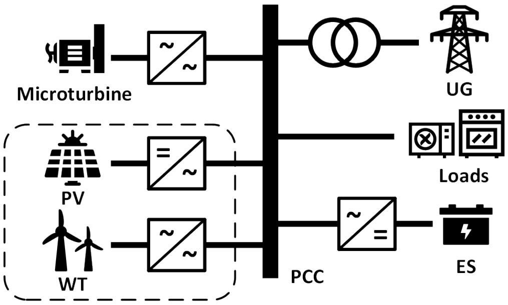

Fig. 1. Microgrid structure.

fixed loads (PL) and dispatchable loads (PD). The schematic diagram of the microgrid is depicted in Fig. 1. Generally, the microgrid can operate either in the grid connected mode or in the islanded mode depending on system configurations and end users' requirements.

The optimization problem herein aims to determine an optimal power dispatch for all controllable units within the microgrid for a predefined finite time horizon $\mathcal{T}$, such that the total operational and maintenance cost F_{MG} is minimized by knowing the prediction of time-of-use (TOU) electricity prices, RES outputs and loads. Due to varying prediction values, the proposed dispatch must be able to optimize the system operation while satisfying power balance requirements in the presence of uncertainties. In other words, the power dispatch should be optimized so that any stochastic variation is properly accommodated.

The optimization model with the constraints of different components in the microgrid can be formulated as follows:

$$F: \min \sum_{t\in\mathcal{T}} \{C_{UG}^t + C_{MT}^t + C_{FC}^t + C_{ES}^t\} \tag{1}$$

where

$$C_{UG}^t = c_B P_{UG_B}^t + c_S P_{UG_S}^t \tag{2}$$

$$C_{MT}^t = a_{MT} {P_{MT}^t}^2 + b_{MT} P_{MT}^t + c_{MT} \tag{3}$$

$$C_{FC}^t = b_{FC} P_{FC}^t \tag{4}$$

$$C_{ES}^t = a_{ES} {P_{ES}^t}^2 \tag{5}$$

Subject to:

1. Power balance:

$$P_{UG}^t + P_{MT}^t + P_{FC}^t + P_{ES}^t + P_{PV}^t + P_{WT}^t = P_L^t + P_D^t \tag{6}$$

2. Energy storage (ES):

$$P_{ES}^t = P_{ES_D}^t \eta_{ES} + P_{ES_C}^t / \eta_{ES} \tag{7}$$

$$0 \le P_{ES_D}^t \le b_{ES}^t P_{ES}^{max} \tag{8}$$

$$(1 - b_{ES}^t) P_{ES}^{min} \le P_{ES_C}^t \le 0 \tag{9}$$

$$E_{ES}^{t-1} + P_{ES}^t \Delta t = P_{ES}^t \tag{10}$$

$$\theta_{ES}^{min} E_{ES}^R \le E_{ES}^t \le \theta_{ES}^{max} E_{ES}^R, \ \forall t \in \mathcal{T} \tag{11}$$

3. Utility grid (UG):

$$P_{UG}^t = P_{UG_B}^t + P_{UG_S}^t \tag{12}$$

$$b_{UG}^t P_{UG}^{min} \le P_{UG_S}^t \le 0 \tag{13}$$

$$0 \le P_{UG_B}^t \le b_{UG}^t P_{UG}^{max} \tag{14}$$

4. Micro turbine (MT):

$$b_{MT}^t P_{MT}^{min} \le P_{MT}^t \le b_{MT}^t P_{MT}^{max} \tag{15}$$

5. Fuel Cell (FC):

$$0 \le P_{FC}^t \le P_{FC}^{max} \tag{16}$$

6. Dispatchable loads (PD):

$$0 \le P_D^t \le P_D^{max} \tag{17}$$

$$C_D^{t-1} - P_D^t \Delta t = C_D^t \tag{18}$$

The optimization objective is to minimize the total operational cost within $\mathcal{T}$ including various cost functions in (2)-(5). (2) denotes the grid-tied electricity tariff, in which the buying price $c_B(t)$ is higher than the selling price $c_S(t)$ to prevent energy arbitrage from the market; (3) denotes the operational cost of MT presented with a quadratic function; (4) denotes the fuel cost of FC and; (5) denotes the operation and maintenance cost. Note that the RES investment cost is not considered since it should be settled in the planning stage. Multiple constraints are presented in (6)-(18) for different components in the microgrid, including UG, ES, MT, FC and PD. The above optimization problem is a mixed-integer quadratic programming (MIQP) model, and such the non-strictly convex model can be solved efficiently by various commercial solvers.

B. Compact Form of Multi-stage Stochastic Optimization

Following the multi-stage scenario-based stochastic program [15], the above optimization model can be formulated into a compact mathematical form, which can be written as follows :

$$\begin{aligned} F_T : \min_{v,w} \ & \sum_{t\in\mathcal{T}} E[\, f_t(v_t, w_t, u_t) \,] \\ \text{s.t. } & G(v_t, w_t, s_{t-1}, s_t, u) = 0, \ \forall \ t \in \mathcal{T} \\ & H(v_t, w_t, s_{t-1}, s_t, u) \le 0, \ \forall \ t \in \mathcal{T} \\ & v_t \in V_t, \ w_t \in W_t, \ s_t \in S_t, \ \forall \ t \in \mathcal{T} \\ & u_t \in \{U_t : (f_t, V_t) \ \forall \ t \in \mathcal{T}\} \end{aligned} \tag{19}$$

where $v_t = \{P_{UG}^t, P_{MT}^t, P_D^t\}$ denotes the *here-and-now* decisions including power dispatches of UG, MT and DL. Such those variables must be made before the uncertain data at time t is known. In the meantime, $w_t = \{P_{FC}^t, P_{ES}^t\}$ denotes the *wait-and-see* decisions including power dispatches of ES and FC to fill up the prediction deviations. $s_t = \{E_{ES}^t, C_D^t\}$ denotes the state variables including the SOC of ES and available capacity of DL. $u_t = \{P_{PV}^t, P_{WT}^t, P_L^t\}$ denotes the scenario instance in the set U_t at t including power of WT, PV and fixed loads. G and H are combination of equality and inequality constraints in (6)-(18), respectively, in which only the state variable set s_{t-1} in last time interval are coupled. Furthermore, $E[f_t(v_t, w_t, u_t)]$ in the objective function is the expected optimal value at t under the succeeding scenario set U_t.

III. Decentralized ADMM based on Scenario Tree

In this section, a temporal decentralized algorithm is proposed for the aforementioned multi-stage stochastic problem F_T, in which ADMM [11] is implemented based on the scenario tree structure. With the linkages established by the method of multipliers [16], the original large-scale convex objective functions are decomposed into a series of smaller subproblems with small variables sets that can be efficiently solved iteratively, leading to a global convergence with short iteration times [17], [18].

A. Scenario Tree Generation

It is expected that the prediction error widens as the forecast horizon increases due to the uncertainty on variation trends [19]. To accommodate a multi-stage stochastic programming model, the scenario tree is useful to approximate the stochastic process evolving with time. On the other hand, the computational burden of the model can be alleviated by reducing the number of scenarios, while the accuracy must be maintained. In this paper, the tree generation algorithm based on the backward scenario reduction [20] is implemented to create a subset $\mathcal{M}$ to be the set of M nodes in the scenario tree, and assign new probabilities $\mathrm{P} = \{p_m : m \in \mathcal{M}\}$ from the initial scenario set for $\mathcal{M}$.

We denote $\mathcal{F}_m$, $\mathcal{C}_m$ to be the root and leaf set of node m, respectively. $\mathcal{F}_m$, $\mathcal{C}_m$ can be $\emptyset$ if m is the top root or the last leaf node in the scenario tree. t_m is denoted to be the index of time interval in which node m is located. Indexed by the node set $\mathcal{M}$ in the scenario tree, the optimization problem (19) can be re-organized as:

$$F_M : \min_{v,w} \sum_{m\in\mathcal{M}} p_m\, f_m(v_m, w_m, u_m) \tag{20}$$

$$\text{s.t. } G(v_m, w_m, s_m, s_m^f, u_m) = 0 \tag{21}$$

$$H(v_m, w_m, s_m, s_m^f, u_m) \leq 0 \tag{22}$$

$$v_m \in V_m,\ \ w_m \in W_m \tag{23}$$

$$s_m^f \in S_m^f,\ \ s_m \in S_m \tag{24}$$

$$u_m \in U_m,\ \ \forall\ m \in \mathcal{M} \tag{25}$$

$$v_m^c = v_n,\ \ \forall n \in \mathcal{C}_m,\ \ m \in \mathcal{M} \tag{26}$$

In (26), we introduce an auxiliary set v_m^c to manifest the coupling requirements on *here-and-now* decision variables, since their values must be determined before uncertain scenarios are realized at each time interval. It is also observed from F_M that both s_m^f in its root $\mathcal{F}_m$ and v_m^c in its leaf $\mathcal{C}_m$ are needed to serve as local information for each individual node $m \in \mathcal{M}$.

B. Decentralized Algorithm with ADMM

For the sake of brevity, we rewrite the formulation (20) as follows:

$$F_M :\ \min_{x_m, x_m'} \sum_{m\in\mathcal{M}} p_m\, f_m(x_m, x_m') \tag{27}$$

$$\text{s.t. } G(x_m) = 0,\ \forall\ m \in \mathcal{M} \tag{28}$$

$$H(x_m) \leq 0,\ \forall\ m \in \mathcal{M} \tag{29}$$

$$x_m = \{v_m, v_m^c, s_m, s_m^f\},\ \forall\ m \in \mathcal{M} \tag{30}$$

$$x_m' = \{w_m\},\ \forall\ m \in \mathcal{M} \tag{31}$$

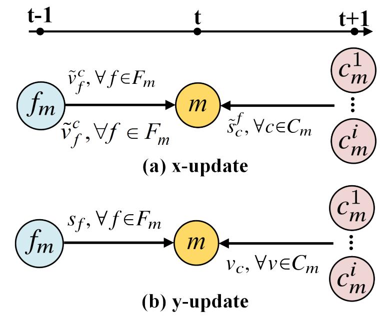

Fig. 2. Data exchange between adjacent nodes.

To convert the original multi-stage stochastic problem into multi-block distributed optimization structure, we define the auxiliary set y_m as the exact replica of the original variable set x_m, and their associated multipliers Λ_m as follows:

$$y_m = \{\tilde{v}_m, \tilde{v}_m^c, \tilde{s}_m, \tilde{s}_m^f\} \tag{32}$$

$$\lambda_m = \{\lambda_{v_m}, \lambda_{v_m^c}, \lambda_{s_m}, \lambda_{s_m^f}\} \tag{33}$$

$$y_m = x_m \tag{34}$$

Note that y_m in (32) does not include any *wait and see* variables in x_m'. The augmented Lagrangian in the scaled form for F_M can be formulated as follows:

$$\mathcal{L}_m = \sum_{m\in\mathcal{M}} \left\{p_m f_m + \frac{\rho}{2}\|x_m - y_m + \lambda_m\|_2^2\right\} \tag{35}$$

The ADMM algorithm solves (35) iteratively for each node m in kth iteration as follows:

$$x_m^{k+1} = \underset{x_m}{\operatorname{argmin}} \left\{p_m f_m + \frac{\rho}{2}\|x_m - y_m^k + \lambda_m^k\|_2^2\right\} \tag{36}$$

$$y_m^{k+1} = \underset{y_m}{\operatorname{argmin}} \left\{\frac{\rho}{2}\|x_m^{k+1} - y_m + \lambda_m^k\|_2^2\right\} \tag{37}$$

$$\lambda_m^{k+1} = \lambda_m^k + (x_m^{k+1} - y_m^{k+1}) \tag{38}$$

Fig. 2 illustrates the sequential procedure of data exchanges between adjacent nodes in x and y updates. in each x-update, the node m acquires $\tilde{v}^c$ from its root F_m and $\tilde{s}^f$ from its leaves set C_m from the optimal values in last y-update. Likewise, in each y-update, the node requires s^f from its root F_m and v^c from its leaves set C_m in last x-update. At last, the multiplier set λ_m is to be updated according to optimal results in the current iteration.

To check the overall convergence of the algorithm, we denote r^k and s^k as the residuals in kth iteration to check the primal and dual feasibility. They are expressed as follows, respectively:

$$r^k = \|x^k - y^k\|_2 \tag{39}$$

$$s^k = \rho\|y^k - y^{k-1}\|_2 \tag{40}$$

Since there is no general rule for the stopping criteria, it is empirical to recognize the ADMM reaches optimality when both r^k and s^k are within the tolerance of $10^{-3}\sqrt{K}$ [11], where K is the number of iterations.

We further implement the proposed temporal ADMM algorithm in a predictive fashion to get optimal scheduling in the

Initialize $j=1$
for $j=1$ **to** N, **do**
  Generate the scenario tree M_j for $\mathcal{T}_j$ according to the selected confidential level, where $t=j, \ \ldots, \ j{+}T$.
  repeat
    for $m \in \mathcal{M}$ **do**
      Solve temporal ADMM by (36)-(38).
    end for
  until Residuals in (39)-(40) are less than stopping criteria.
  Determine v_t by optimal value obtained by ADMM.
  On arrival of the forecast data (RES and loads), determine w_t and s_t.
end for

Fig. 3. Optimization procedure with rolling horizon.

TABLE I
MICROGRID CHARACTERISTICS

Parameter	Value (unit)	Parameter	Value
UG		**ES**	
P_{UG}^{max} (kW)	100	P_{ES}^{max} (kW)	40
P_{UG}^{min} (kW)	-100	P_{ES}^{min} (kW)	-40
MT		η	0.9
P_{MT}^{max} (kW)	100	E_{ES}^{R} (kWh)	150
P_{MT}^{min} (kW)	15	E_{ES}^{0} (kWh)	75
a_{MT} (\$/kWh^2)	$5e^{-4}$	θ_{ES}^{max}	0.95
b_{MT} (\$/kWh)	$2e^{-1}$	θ_{ES}^{min}	0.05
c_{MT} (\$)	0	a_{ES} (\$/kWh^2)	0.12
FC		**DL**	
P_{FC}^{max} (kW)	0	P_{D}^{max} (kW)	0
P_{FC}^{min} (kW)	40	P_{D}^{min} (kW)	-60
b_{FC} (\$/kWh)	0.5	C_{D}^{0} (kWh)	300

rolling horizon $\mathcal{T}$ within a predefined time length N (typically, 24h) [21]. The overall procedure is illustrated in Fig. 3. At current time t, the optimal scheduling is carried out by the proposed ADMM for the entire upcoming horizon, whereas only the dispatch decisions v_t within the present time interval will be committed before realization of the uncertain data U_t., and w_t is then made together with after the real data of RES and loads are obtained. Afterwards, the state variables s_t are updated regarding the full dispatch in time t, and the same scheduling will be executed for $t+1$ until the rolling horizon eventually ends.

IV. CASE STUDY

A. Input Data and Simulation Setting

In this section, the proposed temporal decentralized algorithm is demonstrated in a microgrid, whose specifications are detailed in TABLE. I. For uncertainty modeling, it is assumed the forecast error of loads and renewables unfolds with time increasingly, that the mean value is exactly the forecast and the stand deviation varies progressively from 5% at present to 15% at the end of the prediction horizon. The initial number of sampled scenarios is 50, covering 95% of the confidential level for the whole distribution. It is also reasonably assumed that the forecast values of WT, PV and loads are independent and irrelevant with each other.

The buying and selling electricity prices in 24 hours are depicted in Fig. 4. The rolling horizon N and scheduling horizon T are set to be both 24, respectively. The entire scheduling with the proposed algorithm in a rolling fashion

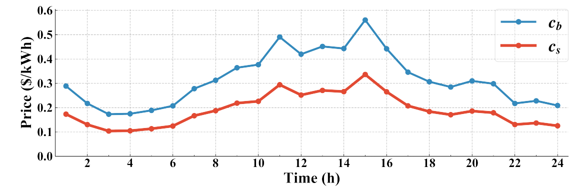

Fig. 4. Electricity price in 24 hours.

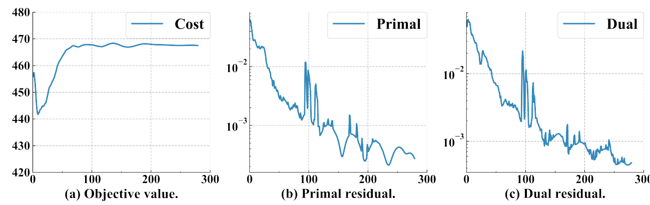

Fig. 5. Convergence of the decentralized algorithm.

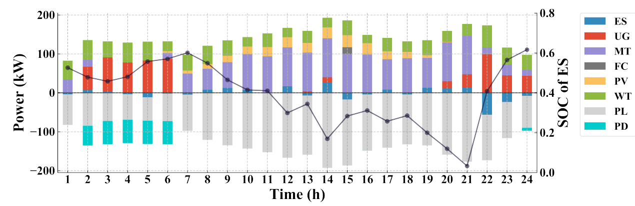

Fig. 6. Optimal energy scheduling and SOC change of ES in microgrids.

is conducted in the Python platform on the PC with 3.10 GHz CPU and 8.0 GB RAM. The commercial solver Gurobi [22] is used to solve the mathematical models in all cases.

B. Results and Discussion

1) Convergence of ADMM: Fig. 5 illustrates the evolution of objective value as well as residual properties. The optimal value in the single epoch can be obtained within 300 iterations, when the stopping criteria of primal and dual residuals satisfied (less than $10^{-3}\sqrt{K}$). Note that the proposed algorithm does choose the typical initial values; it is suggested in [11] that the performance could be further potentially improved by setting the initialization with reasonable values for decision variables that are approximately closed to the optimal solution.

2) Energy Scheduling: Fig. 6 shows the results of the optimal energy scheduling by the proposed decentralized algorithm and two-stage stochastic programming by comparison. It is indicated the controllable units are dispatched according to their costs and operational constraints, that UG only provides power when the electricity price is relatively low, whereas the MT supplies the majority of loads in rest of time intervals. On the other side, PD is also scheduled mostly at hour 2-5 as the reversed power source at low electricity price ranges. In the entire time horizon, FC only provide little power at hour 15 since ES has the full capability to support stochastic errors by inaccurate forecast at most times. ES responds the high price signals effectively as well, since the SOC of ES always ranges with in the desired region even it supplies most of power uncertainties and forecast errors in the prediction.

TABLE II
COMPARISON ON DIFFERENT ALGORITHMS

Type	deterministic	two-stage SO	proposed
Scheduled cost ($)	472.56	489.68	488.44
Committed cost ($)	523.86	501.79	497.22
Tol. comp. time (s)	12.33	129.48	196.33
Ave. comp. time (s)	0.51	6.02	7.96

3) Comparison with Other Methods: To evaluate the performance of the proposed decentralized algorithm, the deterministic optimization with the rolling horizon and the two-stage stochastic optimization based on Benders decomposition [23] are conducted as the comparative cases.

The scheduled and committed cost, total and average computation time of all three cases are presented in TABLE II. It is shown that the deterministic method achieves the minimal scheduled cost, however its committed cost is the highest due to its poor capability of dealing with uncertainties. On the contrary, the proposed decentralized ADMM algorithm achieves a higher scheduled operational cost, while its committed cost exposed to the realization of uncertainties is the lowest among three cases. By comparing with the optimal value using two-stage based method as the benchmark for stochastic optimization, it is clearly indicated the proposed algorithm has achieved a better performance.

As for computation results, it is expected that the proposed algorithm needs the longest computation time, since the optimization procedure of ADMM is exhaustive for all nodes in the scenario tree to reach the convergence. However, these values of the proposed algorithm are very competitive to those two-stage based method, in which the iterative calculation is also applied. Considering there is always a trade-off between optimal gaps and computational complexity, such the comparative result in terms of computation time can be considered to be effective since the energy scheduling for the microgrid is determined hourly.

V. CONCLUSION

In this paper, a novel temporal ADMM decentralized framework based on scenario tree is proposed for stochastic optimization in microgrids. Minimization of the total operational cost considering intermittency of RES as well as uncertainty of loads has been achieved. The mathematical model of the optimization problem is formulated as a multi-stage MIQP stochastic program. By making the decomposition in the scenario tree, each node solves its local convex problem and exchange information with adjacent nodes iteratively. Simulation studies with a local microgrid successfully prove the fast convergence of the temporal decentralized ADMM approach, and the optimized results shows that the proposed algorithm significantly outperforms other mathematical methods such as model predictive control and two-stage stochastic programming algorithms.

ACKNOWLEDGMENT

This research is supported by the National Research Foundation, Prime Minister's Office, Singapore under the Energy Innovation Research Programme (EIRP) Energy Storage Grant Call and administrated by the Energy Market Authority (NRF2015EWT-EIRP002-007).

REFERENCES

[1] J. R. Birge and F. Louveaux, *Introduction to stochastic programming*. Springer Science & Business Media, 2011.

[2] J.-P. Watson and D. L. Woodruff, "Progressive hedging innovations for a class of stochastic mixed-integer resource allocation problems," *Comp. Manag. Science*, vol. 8, no. 4, pp. 355–370, 2011.

[3] J. M. Mulvey and H. Vladimirou, "Applying the progressive hedging algorithm to stochastic generalized networks," *Annals of Oper. Res.*, vol. 31, no. 1, pp. 399–424, 1991.

[4] C. Ju, P. Wang, L. Goel, and Y. Xu, "A two-layer energy management system for microgrids with hybrid energy storage considering degradation costs," *IEEE Trans. Smart Grid*, 2017.

[5] A. Parisio, E. Rikos, and L. Glielmo, "A model predictive control approach to microgrid operation optimization," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 1813–1827, 2014.

[6] Y. Xu, Z. Y. Dong, R. Zhang, and D. J. Hill, "Multi-timescale coordinated voltage/var control of high renewable-penetrated distribution systems," *IEEE Trans. Power Syst.*, vol. 32, no. 6, pp. 4398–4408, 2017.

[7] C. Li, Y. Xu, X. Yu, C. Ryan, and T. Huang, "Risk-averse energy trading in multienergy microgrids: a two-stage stochastic game approach," *IEEE Trans. Indus. Inf.*, vol. 13, no. 5, pp. 2620–2630, 2017.

[8] M. Asensio and J. Contreras, "Stochastic unit commitment in isolated systems with renewable penetration under cvar assessment," *IEEE Trans. Smart Grid*, vol. 7, no. 3, pp. 1356–1367, 2016.

[9] A. B. Philpott and Z. Guan, "On the convergence of stochastic dual dynamic programming and related methods," *Oper. Res. Letters*, vol. 36, no. 4, pp. 450–455, 2008.

[10] P. Falugi, I. Konstantelos, and G. Strbac, "Planning with multiple transmission and storage investment options under uncertainty: A nested decomposition approach," *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 3559–3572, 2018.

[11] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein *et al.*, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine learning*, vol. 3, no. 1, pp. 1–122, 2011.

[12] H. Wang and J. Huang, "Incentivizing energy trading for interconnected microgrids," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 2647–2657, 2018.

[13] T. Erseghe, "Distributed optimal power flow using admm," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2370–2380, 2014.

[14] G. Chen, C. Li, and Z. Dong, "Parallel and distributed computation for dynamical economic dispatch," *IEEE Trans. Smart Grid*, vol. 8, no. 2, pp. 1026–1027, 2017.

[15] S. Ahmed, A. J. King, and G. Parija, "A multi-stage stochastic integer programming approach for capacity expansion under uncertainty," *J. Global Optim.*, vol. 26, no. 1, pp. 3–24, 2003.

[16] N. Parikh, S. Boyd *et al.*, "Proximal algorithms," *Foundations and Trends® in Optimization*, vol. 1, no. 3, pp. 127–239, 2014.

[17] W. Deng, M.-J. Lai, Z. Peng, and W. Yin, "Parallel multi-block admm with o (1/k) convergence," *J. Sci. Comput.*, vol. 71, no. 2, pp. 712–736, 2017.

[18] C. Chen, B. He, Y. Ye, and X. Yuan, "The direct extension of admm for multi-block convex minimization problems is not necessarily convergent," *Math. Program.*, vol. 155, no. 1-2, pp. 57–79, 2016.

[19] G. E. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, *Time series analysis: forecasting and control*. John Wiley & Sons, 2015.

[20] N. Growe-Kuska, H. Heitsch, and W. Romisch, "Scenario reduction and scenario tree construction for power management problems," in *Power Tech Conf. Procc, 2003 IEEE Bologna*, vol. 3. IEEE, 2003, pp. 7 pp. Vol.3–.

[21] Y. Zhang, R. Wang, T. Zhang, Y. Liu, and B. Guo, "Model predictive control-based operation management for a residential microgrid with considering forecast uncertainties and demand response strategies," *IET Gen., Trans. & Distr.*, vol. 10, no. 10, pp. 2367–2378, 2016.

[22] L. Gurobi Optimization, "Gurobi optimizer reference manual," 2018. [Online]. Available: http://www.gurobi.com

[23] J. F. Benders, "Partitioning procedures for solving mixed-variables programming problems," *Num. Math.*, vol. 4, no. 1, pp. 238–252, 1962.